Stability problem and simulation of interaction of the multidimensional NLS solitons in non-uniform and nonstationary media

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Abstract

Investigation of dynamics of multidimensional electromagnetic (EM) waves in a plasma, such as 2D and 3D envelop solitons, is very actual problem. The interaction sufficiently changes the characteristics of the waves and background EM field in the region of interaction. Problem of the dynamics and stability becomes more complicated if it is necessary to take into account an influence of different dispersive and nonlinear inhomogeneities and nonstationary parameters of medium on the soliton structure and evolution. In this case the problem reduces to the generalized nonlinear Schrodinger (GNLS) equation for the amplitude of the EM field with coefficient functions having spatial and temporal inhomogeneities. The analysis of stability of the multidimensional GNLS solitons was based on the method of study of transformational properties of the Hamiltonian of the system developed by authors earlier for the BK class of the equations. As a result we have found the conditions of existence of the multidimensional stable GNLS soliton solutions. At simulation the Fourier splitting method for the GNLS equation was used taking into account the inhomogeneities of coefficient functions of the equation. Implicit scheme of finite-difference method was used for investigation of soliton propagation in non-uniform and nonstationary medium. Numerical modeling showed that inhomogeneity of medium changes the amplitudes of solitons and nonlinear EM waves, their velocities of propagation, their quantity that is caused by their nonelastic interaction in inhomogeneous medium. Nonstationary medium changes a form of impulse and affects its spectral features. Changes of modulation of the parameters of medium make possible variation of character of nonelastic interaction at solitons attraction-repulsion.

Keywords: Nonlinear Waves, Multidimensional Solitons, NLS Equation, Non-uniform and Nonstationary Media, Theory, Numerical Simulation

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Prof. Oleg Kharshiladze is associated professor at physics department of Iv. Javaxishvili Tbilisi State University. He is involved in international scientific group, working on analytical and numerical analysis of ionospheric and magnetospheric processes (turbulence, shear flows, BBF and others).

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Theory and modeling of nonlinear wave processes and solitons in complex dispersive media
The propagation of a soliton pattern through one-dimensional medium with weakly disordered dispersion is considered. Solitons, perturbed by this disorder, radiate. The emergence of a long-range interaction between the solitons, mediated by the radiation, is reported. Basic soliton patterns are analyzed. The interaction is triple and is extremely sensitive to the phase mismatch and relative spatial separations within the pattern. This phenomenon is a generic feature of any problem explaining adiabatic evolution of solitons through a medium with frozen disorder. Standard models of dynamical media are based on partial differential equations, typical examples being the nonlinear Schrödinger (NLS) equation for the mean-field complex wave function $\hat{\psi}(x, y, z, t)$ in atomic Bose-Einstein condensates (BECs; in that case, the NLS equation is usually called the Gross-Pitaevskii equation (GPE) \cite{1}), and the NLS equation for the envelope amplitude of the electromagnetic field in optical media \cite{2}. In the scaled form, the NLS equation is $i\psi_t = -(1/2)\nabla^2\psi + g|\psi|^2\psi + U(x, y, z)\psi$, (1). The 1D continuous NLS equation without the external potential and with either sign of the nonlinearity, $g$, is integrable by means of the inverse-scattering transform \cite{20–23}, although it is nonintegrable in the 2D and 3D geometries. The interaction between two solitons is represented by the solution of the NLS equation which corresponds to two. discrete eigenvalues. $z_1, z_2$ is given by $|a_1| \cosh (a_1 + iq_1) \exp (iq_1)$. Solitons is set at 3. The different profiles set for simulation parabolic, multi parabolic and hyperbolic profiles depending upon the value of potential set in simulation code. The dispersion parameter $b$ is set at $-2 \text{ ps}^2 / \text{ km}$ simulation while pulse width is set at $1 \text{ ps}$. The potential strength is set at 10.

(a) Use Lyapunov stability analysis methods to give a precise statement and a proof of the above argument. (b) System 13.10 is usually solved numerically by the discrete-time system $x(k) = \text{method}$: if the linearized model at an equilibrium point is asymptotically stable, then this equilibrium point of the nonlinear system is asymptotically stable. (We shall actually only consider an equilibrium point at the origin, but the approach can be applied to any equilibrium point, after an appropriate change of variables.)