Activities and Findings

Research and Education Activities:
The research program under this grant was completed with considerable success, resulting in more than 30 research publications, listed in the publications section, and moreover, several students were supported by this grant and two of them have completed their Ph.D's while supported by this grant.

Throughout the time they were supported by this grant, Carlen and Loss collaborated on problems in kinetic theory and functional analysis. In one of the highlights of this work Carlen and Loss, together with Jeff Geronimo, computed the exact spectral gap in the Kac model for physical collisions -- momentum and energy conserving -- in three dimensions [CL6]. The methodology builds on the previous work of Carlen, Carvalho and Loss, but introduced new techniques for using more detailed information about a kinematical 'correlation operator' to get bounds
on the spectrum of the generator of the Kac walk.

The research on the correlation operator arising in the Kac model has led to interesting unanticipated developments: Carlen, Loss and Geronimo have written a paper giving simple, completely elementary proofs of Gasper's and Bochner's theorem on the Markov sequence problems for the Jacobi polynomials [CL35], and this has lead to further developments with more work in progress.

Findings:
Loss made contributions to a variety of fields. Jointly with Baker and Stolz he characterized the minimizing energy configurations in the random displacement model and showed that in two and higher dimensions these configurations are essentially unique [L11, L26]. This work is significant for two reasons. For one it introduces a new technique for analyzing the lowest Neumann eigenvalue as a function of the position of the potential (bubbles tend to the boundary) and, moreover, it is substantial progress towards proving localization in the random displacement model.

In [L2], which is joint work with Miyao and Spohn, the existence of ground states of moving atoms in non-relativistic quantum electrodynamics (QED) is proved. The significance of this paper is that the result holds under natural, non-technical conditions. All previous results assumed that the fine structure constant and the ultra-violet cutoff are sufficiently small. Likewise in [L27], again work with Miyao and Spohn, it is shown that various QED-systems have degenerate eigenvalues once the spin of the electron is included. This follows by a careful analysis of the Kramers degeneracy. It explains why, without and external magnetic field, there is no level splitting in the spin.

In [L18, L21] which is joint work with Dolbeault and Esteban, the Dirac hydrogen atom interacting with a strong magnetic field is analyzed in depth. In particular there is a critical size of the magnetic field at which the lowest eigenvalue penetrates the lower continuum. Curiously, the lowest Landau level approximation does not yield a very good result for the size of this critical field.

Carlen has made several contributions in dissipative evolutions. In a paper with A. Blanchet and J. A. Carrillo [C34], the long time behavior of critical mass solutions of the Patlak-Keller-Segal system is studied, and basis of attraction are found for the steady state solutions. These steady state solutions are minimizers of the logarithmic Hardy-Littlewood-Sobolev inequality, and are also stationary solutions of a certain porous medium equation. The analysis in this work turns on a surprising relation between the Patlak-Keller-Segal system and the porous medium equation, and involves the development of a number of new functional inequalities. The paper makes use of optimal mass transportation techniques, and extends existing methods to work in what is a challenging 'critical' setting. Other contributions to the use of optimal mass transportation in variation problems are made in the paper [C11] by Carlen, Carvalho, Esposito, Lebowitz and Marra.

Together with Carvalho and Lu, Carlen has completed an investigation [C29] of the spatially homogeneous Boltzmann equation with soft potentials. This paper provides much improved bounds on both the growth of moments, and the rate of convergence to equilibrium for soft potentials, and introduced a range of new techniques. Soft potentials are physically relevant, but technically much more difficult to deal with than hard potentials.

While supported by this grant, Carlen has continued to work on optimal matrix inequalities. Results obtained in this are, in joint work with Lieb, are in [C19] and in [C22] which develops non-commutative analogs of Brascamp-Lieb inequalities. This paper uses an extension to the non-commutative setting of recent results of Carlen and Cordero [C25], also supported by this grant.

All citations in these sections are listed in the publications section.

Training and Development:

Outreach Activities:


Books or Other One-time Publications


Web/Internet Site

URL(s):
www.math.gatech.edu/~loss and www.math.rutgers.edu/~carlen/

Description:
Besides publication in refereed journals, our work is disseminated through the archive http://arxiv.org/ and on the PIs URLs listed above. All of these papers acknowledge NSF support.

Other Specific Products

Contributions within Discipline:

Contributions to Other Disciplines:

Contributions to Human Resource Development:
Loss's student Craig Sloane is progressing towards his thesis, with papers in progress as noted above, as is Amit Einav. Carlen's student Suleyman Ulusoy graduated with his Ph.D. in August 2007, spent two years a postdoctoral fellow at the Centre of Mathematics for Application at the University of Oslo, and is now that the University of Maryland in a second postdoctoral position. Two papers of his, one joint with Carlen, were written while Ulusoy was supported by this grant. A second student of Carlen, Jogia Bandyopadhyay, defended her Ph.D. thesis in April 2008, and is now a postdoctoral fellow at the University of Helsinki. Her supported thesis work is published in Comm. Math. Phys., 28 no. 3 (2009) 1065-1086.

Contributions to Resources for Research and Education:

Contributions Beyond Science and Engineering:

Conference Proceedings
Categories for which nothing is reported:

Organizational Partners
Activities and Findings: Any Training and Development
Activities and Findings: Any Outreach Activities
Any Product
Contributions: To Any within Discipline
Contributions: To Any Other Disciplines
Contributions: To Any Resources for Research and Education
Contributions: To Any Beyond Science and Engineering
Any Conference
Variational Methods for Computational Fluid Dynamics. François Alouges and Bertrand Maury. 2. Contents. 1 Models for incompressible fluids 1.1 Some basics on viscous fluid models 1.2 Mathematical framework 1.3 Free in-/outlet conditions 2 In the context of lung modeling, this technical problem will actually disappear, as we shall deal with domain with inlet and outlets, so that this indeterminacy shall not be met. 1.2. MATHEMATICAL FRAMEWORK. 11. celebrated paper by Leray [13], but uniqueness of those weak solutions is an open issue. The two standpoints are related, as uniqueness of smooth solutions holds. In addition, the underlying dynamics associated with such problems, additional qualitative analysis, as well as algorithms, can be found in the book by Nagurney and Zhang (1996): Projected Dynamical Systems and Variational Inequalities with Applications. Professor Anna Nagurney. SCH-MGMT 825 Management Science Seminar. Background. When one considers decision-making, one, typically, has a decision-maker in mind, his/her objective function to be optimized, the decision variables, as well as the constraints that make up the feasible set. Such problems are cast as optimization problems. On the other hand, most researchers have considered this problem from one of two perspectives: maximize a portfolio’s expected return for a given risk or minimize a portfolio’s risk for a given expected return. In this talk, we consider a fractional model with a returns over risk ratio that keeps both expected returns and risk flexible simultaneously. The considered model provides us with the optimal investment portfolio for which the expected returns per unit of risk will be maximized. We demonstrate how to reformulate the basic model as a linear program. Numerical tests demonstrate that the model is no harder